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Zeroing In

Solving Quadratics by Factoring

LEARNING GOALS

In this lesson, you will:

- Solve quadratic equations and functions using factoring.
- Connect the zeros of a function to the *x*-intercepts of a graph.
- Determine the roots of quadratic equations.

KEY TERMS

- Zero Product Property
- Converse of Multiplication Property of Zero
- roots

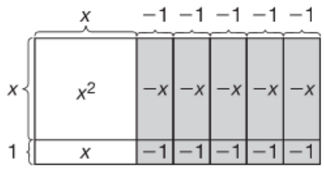
The word *zero* has had a long and interesting history. The word comes from the Hindu word *sunya*, which meant "void" or "emptiness." In Arabic, this word became *sifr*, which is also where the word *cipher* comes from. In Latin, it was changed to *cephirum*, and finally, in Italian it became *zevero* or *zefiro*, which was shortened to *zero*.

The ancient Greeks, who were responsible for creating much of modern formal mathematics, did not even believe zero was a number!

PROBLEM 1 Roots of Quadratic Equations

Recall that a quadratic expression of the form $x^2 + bx + c$ can be factored using an area model, a multiplication table, or the factors of the constant term c. The quadratic expression $x^2 - 4x - 5$ is factored using each method as shown.

- Area model



$x^2 - 4x - 5 = (x - 5)(x + 1)$

- Multiplication table

·	x	-5
x	x^2	$-5x$
1	x	-5

$x^2 - 4x - 5 = (x - 5)(x + 1)$

- Factors of the constant term c

Factors of -5: $-5, 1$ $-1, 5$

Sums: $-5 + 1 = -4$ $5 + (-1) = 4$

$x^2 - 4x - 5 = (x - 5)(x + 1)$

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The **Zero Product Property** states that if the product of two or more factors is equal to zero, then at least one factor must be equal to zero.

If $ab = 0$, then $a = 0$ or $b = 0$.

This is also referred to as the **Converse of the Multiplication Property of Zero**.

1. Use the Zero Product Property to determine the solutions of the quadratic equation $x^2 - 4x - 5 = 0$. Then, check your solutions by substituting back into the original equation.

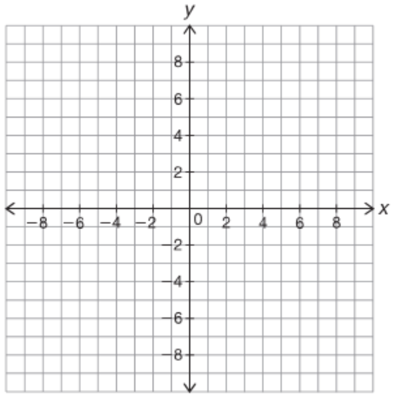
2. Let's examine the quadratic equation $0 = x^2 - 4x - 5$.

a. Graph both sides of the quadratic equation on the coordinate plane shown.

b. Rewrite the equation in factored form.

c. Identify the vertex, x- and y-intercepts, and the axis of symmetry.

- y-intercept:
- x-intercept(s):
- axis of symmetry:
- vertex:



d. Compare the x-intercepts of the equation $y = x^2 - 4x - 5$ to the solutions to Question 1. What do you notice?

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e. Compare the intersections of the two equations you graphed to the solutions to Question 1. What do you notice?

The solutions to a quadratic equation are called *roots*. The **roots** indicate where the graph of a quadratic equation crosses the x-axis. So, roots, zeros, and x-intercepts are all related.

To calculate the roots of a quadratic equation using factoring:

- Perform transformations so that one side of the equation is equal to zero.
- Factor the quadratic expression on the other side of the equation.
- Set each factor equal to zero.
- Solve the resulting equations for the roots. Check each solution in the original equation.

You can calculate the roots for the quadratic equation $x^2 - 4x = -3$.

$$x^2 - 4x = -3$$
$$x^2 - 4x + 3 = -3 + 3$$
$$x^2 - 4x + 3 = 0$$
$$(x - 3)(x - 1) = 0$$
$$(x - 3) = 0 \quad \text{or} \quad (x - 1) = 0$$
$$x - 3 + 3 = 0 + 3 \quad \text{or} \quad x - 1 + 1 = 0 + 1$$
$$x = 3 \quad \text{or} \quad x = 1$$

Check:

$$x^2 - 4x = (3)^2 - 4(3) = 9 - 12 = -3$$
$$x^2 - 4x = (1)^2 - 4(1) = 1 - 4 = -3$$

Determine the roots of each quadratic equation.

3. $x^2 - 8x + 12 = 0$

4. $x^2 - 5x - 24 = 0$

5. $x^2 + 10x - 75 = 0$

6. $x^2 - 11x = 0$

7. $x^2 + 8x = -7$

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8. $x^2 - 5x = 13x - 81$

9. $3x^2 - 22x + 7 = 0$

10. $8x^2 + 2x - 21 = 0$

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PROBLEM 2 More Practice

Calculate the zeros of each quadratic function, or the roots of each quadratic equation, if possible.

1. $f(x) = x^2 - 7x - 18$

2. $f(x) = x^2 - 11x + 12$

3. $f(x) = x^2 + 10x - 39$

4. $2x^2 + 4x = 0$

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5. $\frac{2}{3}x^2 - \frac{5}{6}x = 0$

Be prepared to share your solutions and methods.